On computation of electrostatic field strength at triple junctions

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Abstract - The work examines the problems of calculating the electrostatic field strength near triple junctions, where the boundaries of metal, dielectric and vacuum intersect. It is shown that of much importance is the ratio of opening angles of sectors filled by each medium.

I. Introduction

Calculations of electrostatic field are an essential constituent part of the work on determining the electric strength of equipment. In case of vacuum electric strength in presence of dielectrics, computing the field strength near a triple junction presents a severe difficulty.

There are some aspects of the difficulty of calculating the field near a triple junction. The first one is due to the metallization layer being quite thin as compared to other system sizes. Therefore, the calculation typically assumes the metallization layer to be infinitesimally thin, and solving the problem just proceeds from the condition of constant potential on the metallized surface of dielectric. The long-time application of such approach has shown that, while there are strong doubts as to the magnitude of the field strength at a triple junction, the comparison of equipment of different configuration by the computation results is quite correct when collating them with experimental ones, i.e. the ratio of field strengths in different units of equipment, found in the same manner, can well be of use.

The doubts in thus computed value of the strength magnitude are quite justified. The matter is that such a problem is incorrect from the viewpoint of mathematical physics. For the boundary problem for Poisson's equation to be correct, the boundary must consist of points representable as vertices of pyramids of finite volume, all other points of them lying outside the volume in question ([1]).

II. Refind calculations of the field near triple junction

There was an opinion that the said difficulty was of exclusively technical, computational nature and, provided the actual finite thickness of the metallization layer is taken into account, everything would be all right. To consider the finite thickness, the submodeling technique is used, when an initial rough computation of the field is performed ignoring the finite thickness of the metallization layer and then a small neighborhood of a triple junction is selected, with the boundary values obtained in the rough computation being set on its outer boundary.

The executed series of test calculations have shown that the approach is quite realizable and yields good results for some configurations. At the same time, the calculations based on this technique produce entirely different results for other configurations, including those corresponding to actual geometry of ceramics used. Namely, when the submodeling is used, the field strength at a triple junction is the higher, the finer mesh is applied at the second stage of the submodeling, and obviously tends to infinity.

Although solving real-world problems typically draws on the axisymmetric model, there was an opinion that the said effect of growing field strength could be related with the problem dimensionality. However, the executed test calculations for three-dimensional models failed to change the result – the field strength at a triple junction tends to infinity with the mesh refinement.

It has become clear that the tending of the field strength to infinity reflects an actual singularity of the field at a triple junction and the issue calls for special examination.

III. Singularities of the electric field at a triple junction

Consider a two-dimensional problem. Let sectors with opening angles θ_1 and θ_2 be filled by dielectrics with permittivities ε_1 and ε_2, respectively, and the rest of the space – by conductor.

To find the distribution of the potential in dielectrics, we seek to solve Laplace's equation, which in the polar coordinates r, θ has the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}=0 \qquad (1)$$

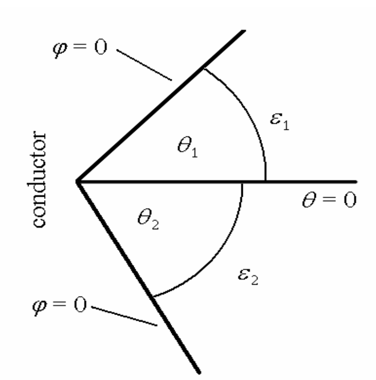

Fig. 1

We direct the polar axis along the interface of the dielectrics (see Fig. 1). The boundaries of the dielectrics correspond to $\theta = \theta_1$ and $\theta = -\theta_2$. Eq. (1) admits separation of variables. The solution in region (1), satisfying boundary condition $\varphi(r, \theta_1) = 0$, has the form

$$\varphi(r,\theta) = \sum_k a_k r^k \sin(k(\theta - \theta_1)) \quad (0 \le \theta \le \theta_1) \qquad (2)$$

The solution in region (2), satisfying boundary condition $\varphi(r, -\theta_2) = 0$, has the form

$$\varphi(r,\theta) = \sum_k b_k r^k \sin(k(\theta - \theta_2)) \quad (0 \le \theta \le -\theta_2) \qquad (3)$$

The continuity of the potential and the continuity of the electric induction vector component, normal to the interface, $D_n = -\frac{1}{r}\frac{\partial \varphi}{\partial \theta}$, at $\theta = 0$ yield the equation for possible values of k:

$$\frac{\tan(k\theta_1)}{\tan(k\theta_2)} = \frac{\varepsilon_1}{\varepsilon_2}, \qquad (4)$$

which is the one to determine parameter k values. Coefficients a_k and b_k must be found from boundary conditions on other (distant) surfaces.

Fig. 2 gives the results of calculations of the least values of k for $\varepsilon_1 = 1$ (vacuum) and $\varepsilon_2 = 9.1$ (ceramics). The level lines for k are shown on $\theta_1\theta_2$ plane. At $k < 1$, the solution exhibits a singularity – the field strength grows without bound at $r \to 0$. The computations allow finding those parameter values where the field singularities appear.

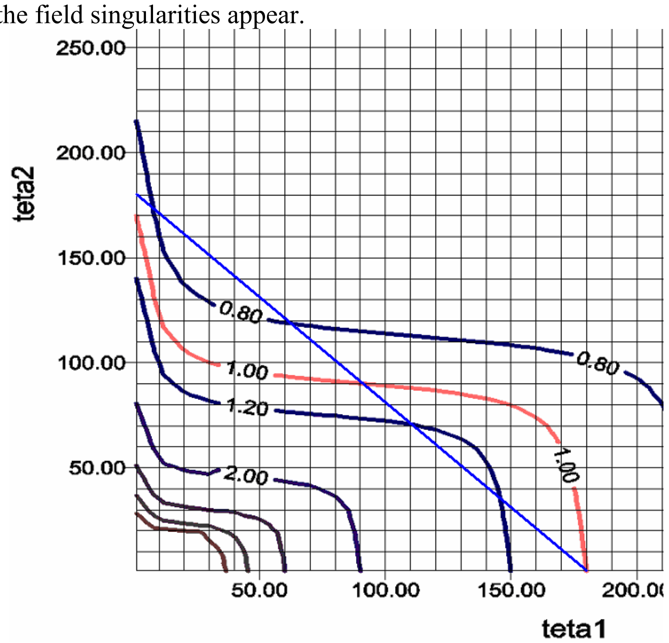

Fig. 2

Consider, for instance, the variant of applying a metal layer to the planar surface of ceramics (Fig. 3, top). In this case, $\theta_2 = 180°$, and, generally speaking, there is no solution without a singularity.

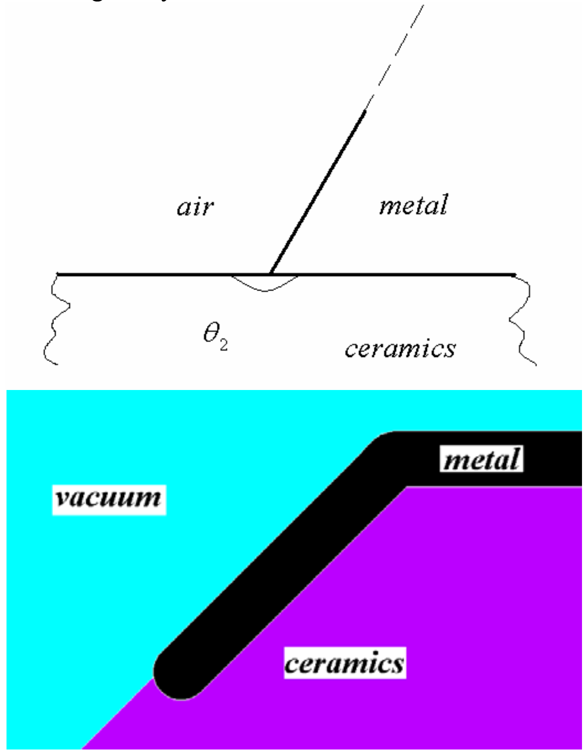

Fig. 3

As a variant of coating when there are no singularities, one can cite the case $\theta_1 + \theta_2 = 180°$, i.e. the dielectrics adjoin the planar conductor surface. Blue color on Fig. 2 marks straight line $\theta_1 + \theta_2 = 180°$. As seen from the figure, with $0 \le \theta_2 \le 90°$ (θ_2 is the angle filled by ceramics), the solution should have no singularity at a triple junction. A possible variant of such coating is shown on Fig. 3, bottom.

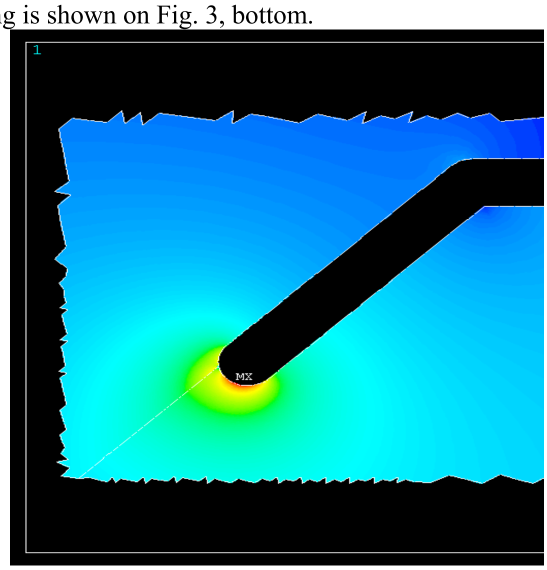

Fig. 4

As seen from the figure 4, where the result of calculation of electric field with ANSYS package is presented, there is indeed no singularity at a triple junction.

Conclusion

The conducted study shows that the configuration of dielectric with a metallization layer should be designed with mandatory taking into account of possible field singularities.

References

[1] Courant R. *Partial Differential Equations.* New York, London, 1962.